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AMC 12 Free Mock Test

25 problems · answer key included · printable edition

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AMC 12 FREE MOCK — INSTRUCTIONS

1. 25 multiple-choice questions. Each has answers A–E; only one is correct.
2. Score = **6** × correct + **1.5** × unanswered. Incorrect = **0**.
3. Scratch paper, ruler, compass, eraser allowed. No calculators, phones, or watches.
4. Figures are not necessarily drawn to scale.
5. You have **75 minutes**.

PROBLEMS

Problem 1.

A bag contains four green balls and five purple balls. Balls are randomly drawn one by one out of the bag until there are none left in the bag. What is the probability that the seventh ball drawn is green?

(A) $\frac{1}{9}$ **(B)** $\frac{4}{9}$ **(C)** $\frac{5}{9}$ **(D)** $\frac{2}{3}$ **(E)** $\frac{7}{9}$

Problem 2.

If $f(x) = 2x + 3$ and $f(g(x)) = x$, what is the value of $g(5)$?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

Problem 3.

Two sides of a non-degenerate triangle are 5 and 9. If the third side is an integer, what is the maximum possible perimeter of the triangle?

(A) 23 **(B)** 25 **(C)** 26 **(D)** 27 **(E)** 28

Problem 4.

Let $p(x)$ be a polynomial $ax^2 + bx + c$ where a is chosen from $\{3, 4, 5, 6\}$, b is chosen from $\{2, 3, 4\}$, and c is chosen from $\{2, 3, 4, 5, 6\}$. How many such polynomials $p(x)$ have at least one rational root?

(A) 0 **(B)** 1 **(C)** 3 **(D)** 5 **(E)** 12

Problem 5.

Four friends, A, B, C, and D, finish a race with no ties.
A says, "I did not finish first."
B says, "I finished third."
C says, "A finished behind D."
D says, "I finished last."
If exactly one person is lying, who finished first?

(A) A **(B)** B **(C)** C **(D)** D **(E)** Cannot be determined

Problem 6.

Three triangles can be joined such that one edge of each triangle coincides with one edge of each of the other triangles. What is true about the three triangles?

(A) The sum of their smallest angles is 180°. **(B)** Exactly one triangle is obtuse.

(C) At least two triangles must be obtuse. **(D)** None of these is necessarily true.

(E) Exactly two triangles are right triangles.

Problem 7.

A baker sells cookies in boxes of a and b, where a and b are relatively prime positive integers and $a < b$. The largest number of cookies that cannot be purchased exactly using a combination of these boxes is 143. What is the minimum possible value of $a + b$?

(A) 27 **(B)** 28 **(C)** 32 **(D)** 42 **(E)** 53

Problem 8.

A committee of 4 people is to be chosen from a group of 6 married couples. The committee must contain exactly one complete married couple. Furthermore, one person on the committee must be named President, and one person (different from the President) named Vice President. If the President and Vice President cannot be married to each other, how many such distinct committee structures can be formed?

(A) 1920 **(B)** 2160 **(C)** 2400 **(D)** 2880 **(E)** 3120

Problem 9.

Let $ABCDEFGH$ be a regular octagon with side length 2. Let X be the intersection of diagonals AC and BE. What is the area of $\triangle ABX$?

(A) $\sqrt{2}-1$ **(B)** $2\sqrt{2}-2$ **(C)** 2 **(D)** $2\sqrt{2}$ **(E)** $4\sqrt{2}-4$

Problem 10.

A ball is to be thrown into one of ten bins, numbered 1 through 10 in that order, standing in a row. The probability of the ball being thrown into bin n is denoted by p_n. The probability the ball is thrown in the first or last bin is p (so $p_1 = p_{10} = p$). The probability of the ball being thrown into bin n for $1 < n < 10$ is given by $p_n = \dfrac{p_{n-1} + p_{n+1}}{2}$. What is p?

(A) $\dfrac{1}{20}$ **(B)** $\dfrac{1}{10}$ **(C)** $\dfrac{1}{9}$ **(D)** $\dfrac{2}{9}$ **(E)** $\dfrac{1}{5}$

Problem 11.

The sum of all real numbers x that satisfy the following equation:

$$\log_{2x}(8) + \log_4(x^2) = \frac{11}{2}$$

can be written in the form $\dfrac{m+\sqrt{n}}{p}$, where m, n, and p are positive integers and n is not divisible by the square of any prime. Find $m+n+p$.

(A) 64 **(B)** 66 **(C)** 68 **(D)** 70 **(E)** 72

Problem 12.

How many five-digit palindromes (numbers that read the same backward as forward, such as 25352) are divisible by 11?

(A) 72 **(B)** 80 **(C)** 82 **(D)** 90 **(E)** 100

Problem 13.

Let a and b be positive integers such that $a, b \leq 10000$ and

$$\operatorname{lcm}(a, b) = 74 \cdot \gcd(a, b)$$

What is the sum of all possible maximum powers of 17 that divide ab?

(A) 6 **(B)** 8 **(C)** 10 **(D)** 12 **(E)** 14

Problem 14.

A parabola has its vertex at $(-3, -3)$. Its focus, a point on the parabola, and a point on the directrix the same distance from the point as the focus, can be connected to form an equilateral triangle of side length 3. What is the positive difference between the roots of the equation represented by the parabola?

(A) 3 **(B)** 6 **(C)** 9 **(D)** 12 **(E)** 15

Problem 15.

Let $f(k) = 1 + (1 - \frac{k}{50})\, i$. Let $\arg(z)$ be the argument of the complex number z in the range $[0, 2\pi)$. What is $\arg(f(0)f(1)f(2) \ldots f(100))$?

(A) 0 **(B)** $\frac{\pi}{4}$ **(C)** $\frac{\pi}{2}$ **(D)** π **(E)** $\frac{3\pi}{2}$

Problem 16.

Let $ABCDEF$ be a regular hexagonal sheet of paper of side length 2. Points C and F are pinned to the ground while A and B are lifted until the trapezoid $ABCF$ is perpendicular to the trapezoid $CDEF$. Points A and E are connected, as are B and D. The volume of the polyhedron contained within the faces AEF, $ABCD$, BCD, $ABCF$, and $CDEF$ can be written in the form $\frac{m\sqrt{p}}{q}$ where p is not divisible by the square of any prime, and m and q are relatively prime positive integers. What is $m + p + q$?

(A) 10 **(B)** 11 **(C)** 12 **(D)** 13 **(E)** 14

Problem 17.

Suppose there are 77 cupcakes and 22 cookies from which you must choose exactly 12 desserts consisting of any amount of cupcakes and cookies (even 12 of one dessert and 0 of the other is permitted). Let N be the number of ways you can do this. What are the last two digits of N?

(A) 16 **(B)** 36 **(C)** 56 **(D)** 76 **(E)** 96

Problem 18.

A right circular hollow cone with radius $\sqrt{3}$ and height 3, without a base, opens upwards, and a sphere of radius 2 is placed inside the cone. A smaller cone is placed inside the section of the sphere, horizontally laying, bounded upwards by the base of the larger cone (if it had a base). The apex of this smaller cone is equidistant from the bottom of the sphere and a point on the circumference of the base of the larger cone. The base of this smaller cone is tangent to the base of the large cone and the sphere's surface below it. The volume of this small cone can be written as

$$(\sqrt{3} - 1)\pi \cdot \sqrt{m - n\sqrt{p}}$$

where m, n, p are positive integers such that p is not divisible by the square of any prime. What is $m + n + p$?

(A) 5 **(B)** 6 **(C)** 7 **(D)** 8 **(E)** 9

Problem 19.

Three distinct positive integers x, y, and z are randomly selected from the set $\{1, 2, 3, \ldots, 50\}$. What is the probability that $x^3 + y^3 + z^3 - 3xyz = 7(x + y + z)$?

(A) $\frac{47}{9800}$ **(B)** $\frac{47}{4900}$ **(C)** $\frac{141}{9800}$ **(D)** $\frac{94}{4900}$ **(E)** $\frac{47}{2450}$

Problem 20.

Let n be the least positive integer such that the equation

$$\lfloor x^2 + 6 \rfloor = nx + 1$$

has the maximum possible number of solutions it can. What is the remainder when n^{100} is divided by 13?

(A) 1 **(B)** 3 **(C)** 4 **(D)** 7 **(E)** 9

Problem 21.

A sequence of real numbers $\{x_n\}$ is defined by $x_1 = 2, x_2 = \sqrt{3} - 1$, and for all integers $n \geq 2$:

$$x_{n+1} = \sqrt{3}x_n - x_{n-1}$$

Evaluate the series:

$$S = \sum_{n=1}^{23} \frac{1}{x_n x_{n+1}}$$

(A) $\frac{1-\sqrt{3}}{2}$ **(B)** $\frac{2-\sqrt{3}}{4}$ **(C)** $\frac{1-\sqrt{3}}{4}$ **(D)** $\frac{\sqrt{3}-1}{4}$ **(E)** $\frac{1+\sqrt{3}}{4}$

Problem 22.

The sum

$$\binom{2027}{0}+\binom{2027}{4}+\binom{2027}{8}+\cdots+\binom{2027}{2024}$$

can be written in the form $2^p - 2^q$ where p and q are positive integers. What is $p+q$?

(A) 3031 **(B)** 3033 **(C)** 3035 **(D)** 3036 **(E)** 3037

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Problem 23.

Let a square S on the Cartesian plane have vertices $(0,0)$, $(2,0)$, $(2,2)$, and $(0,2)$, and let a square T on the Cartesian plane have vertices $(2,4)$, $(4,4)$, $(4,6)$, and $(2,6)$. Square S will undergo a sequence of four transformations, each being either a 45°, 90°, 135°, 180°, 225°, 270°, or 315° rotation clockwise about the point $(0,4)$. What is the probability that at the end of this sequence of transformations, S is mapped to T?

(A) $\frac{285}{2401}$ **(B)** $\frac{292}{2401}$ **(C)** $\frac{300}{2401}$ **(D)** $\frac{308}{2401}$ **(E)** $\frac{315}{2401}$

Problem 24.

Let $P=\overline{r_{100}\overline{r_{99}\ldots\overline{r_1}}}$, where $r_1, r_2, \ldots, r_{100}$ is a permutation of the roots of $x^{100}-1=0$. It can be shown that the value of P depends on a sum $S=v_1+v_2+\cdots+v_{50}$, where $v_1, v_2, \ldots, v_{50}$ are distinct elements chosen from $\{1,2,\ldots,100\}$. The imaginary part of P attains its maximum possible value for exactly n distinct values of S. What is n?

(A) 49 **(B)** 50 **(C)** 99 **(D)** 124 **(E)** 100

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Problem 25.

Let $N = 20!$. A point $P(x, y)$ in the Cartesian plane is called a factorial lattice point if both x and y are integers such that $1 \leq x, y \leq N$. A straight line passing through the origin $(0, 0)$ and at least one factorial lattice point P is drawn. Let K be the total number of distinct lines that can be drawn this way. For each such line L_i (where $1 \leq i \leq K$), let C_i denote the exact number of factorial lattice points that lie on L_i. The sum

$$\sum_{i=1}^{K} (C_i - 1)$$

can be written in the form $m \cdot (n!)^2 + p \cdot n!$ where $n = 20$. What is $m + p$?

(A) 190 **(B)** 209 **(C)** 210 **(D)** 229 **(E)** 256

ANSWER KEY

1. B **2.** A **3.** D **4.** A **5.** E **6.** C **7.** A **8.** C **9.** B **10.** B **11.** C **12.** C

13. A **14.** B **15.** A **16.** D **17.** E **18.** C **19.** A **20.** E **21.** C **22.** E **23.** C

24. E **25.** —